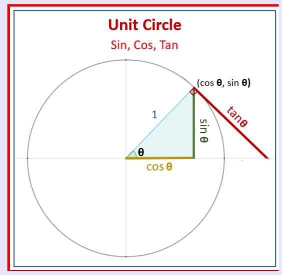
Trigonometry

Lecture 44

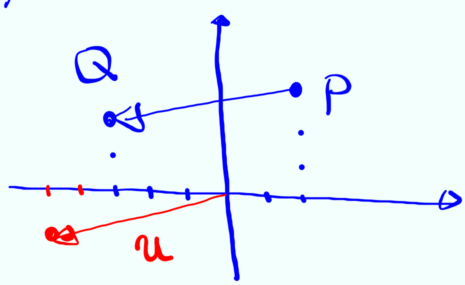


Feb 19-8:47 AM

Ginven $P(2, 3)$, $Q(-3, 2)$

1) Find $\overrightarrow{PQ} = \langle -3-2, 2-3 \rangle$
$= \langle -5, -1 \rangle$

2) Draw $\overrightarrow{PQ}$



3) Draw $u = \overrightarrow{PQ}$ in standard position.

4) Find $|u| = \sqrt{(-5)^2 + (-1)^2} = \sqrt{26}$

Nov 19-10:26 AM

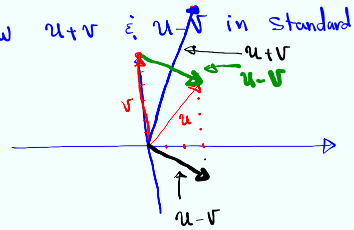
Given $u=\langle 3,4\rangle$, $v=\langle 0,6\rangle$

1) Find $u+v$, $u-v$

$u+v=\langle 3+0,4+6\rangle=\langle 3,10\rangle$

$u-v=\langle 3-0,4-6\rangle=\langle 3,-2\rangle$

2) Draw $u+v$ & $u-v$ in Standard Position



3) $3u-2v$

$=3\langle 3,4\rangle-2\langle 0,6\rangle=\langle 9,12\rangle-\langle 0,12\rangle$

$=\langle 9,0\rangle$

4) $u\cdot v=3\cdot 0+4\cdot 6=\boxed{24}$

5) $|u|=\sqrt{3^2+4^2}=\boxed{5}$ 6) $|v|=\sqrt{0^2+6^2}=6$

7) Find the angle between u and v.

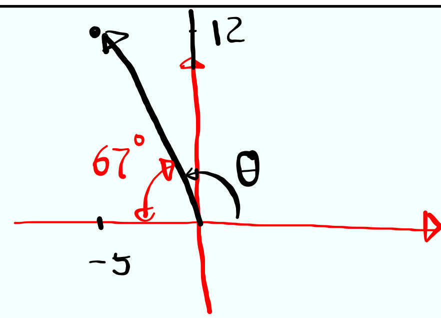
$\cos\theta=\dfrac{u\cdot v}{|u||v|}$ $\cos\theta=\dfrac{24}{5\cdot 6}$ $\cos\theta=\dfrac{4}{5}$

$\theta=\cos^{-1}\left(\frac{4}{5}\right)$

$\theta\approx 37^\circ$

Nov 19-10:31 AM

Given $u=\langle -5,12\rangle$



1) $|u|=\sqrt{(-5)^2+12^2}=\sqrt{169}=13$

2) Find its direction angle θ. $\cos\theta=\dfrac{-5}{13}$

$\sin\theta=\dfrac{12}{13}$

RA $\sin^{-1}\dfrac{12}{13}=67^\circ$

$\theta=180^\circ-67^\circ$

$=113^\circ$

$u=\langle |u|\cos\theta\,,\,|u|\sin\theta\rangle=\langle 13\cos 113^\circ,\,13\sin 113^\circ\rangle$

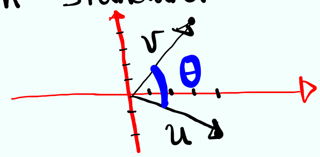
Nov 19-10:41 AM

$u = 4i - 2j \qquad v = 3i + 4j$

1) Draw u & v in standard Position.

$u = \langle 4, -2 \rangle$

$v = \langle 3, 4 \rangle$



2) Find the angle between u & v.

$\cos\theta = \dfrac{u \cdot v}{|u||v|} = \dfrac{4}{\sqrt{20}\,5}$

$u \cdot v = 4(3) + (-2)(4)$
$= 12 - 8$
$= 4$

$|u| = \sqrt{4^2 + (-2)^2} = \sqrt{20}$

$|v| = \sqrt{3^2 + 4^2} = 5$

$\cos\theta = \dfrac{\cancel{4}^{2}}{5 \cdot \cancel{2}\sqrt{5}} \cdot \dfrac{\sqrt{5}}{\sqrt{5}}$

$\cos\theta = \dfrac{2\sqrt{5}}{25} \qquad \theta = \cos^{-1}\left(\dfrac{2\sqrt{5}}{25}\right)$

$\approx 80^\circ$

Nov 19-10:46 AM

Prove $|u|^2 = u \cdot u$

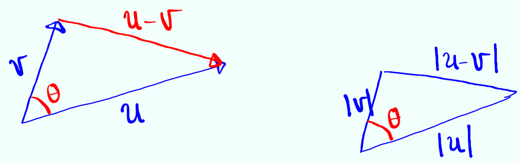
Suppose $u = \langle a, b \rangle$

$|u| = \sqrt{a^2 + b^2}$

$|u|^2 = \left(\sqrt{a^2 + b^2}\right)^2 = a^2 + b^2$ ✓

$u \cdot u = \langle a, b \rangle \cdot \langle a, b \rangle = aa + bb = a^2 + b^2$ ✓

Nov 19-10:54 AM



Law of Cosine

$$|u-v|^2 = |u|^2 + |v|^2 - 2|u||v|\cos\theta$$

$$(u-v)\cdot(u-v) = u\cdot u + v\cdot v - 2|u||v|\cos\theta$$

$$\cancel{u\cdot u} - u\cdot v - u\cdot v \cancel{+v\cdot v} = \cancel{u\cdot u} \cancel{+v\cdot v} - 2|u||v|\cos\theta$$

$$\cancel{-2}\, u\cdot v = \cancel{-2}\,|u||v|\cos\theta$$

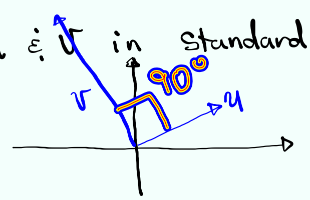
$$u\cdot v = |u||v|\cos\theta$$

$$\boxed{\cos\theta = \frac{u\cdot v}{|u||v|}}$$

Nov 19-10:57 AM

$u = \langle 4, 3\rangle \qquad v = \langle -6, 8\rangle$

1) Draw u & v in standard position.



2) Find $u\cdot v$, $|u|$, and $|v|$

$u\cdot v = \langle 4,3\rangle\cdot\langle -6,8\rangle = -24 + 24 = 0$

$|u| = 5 \quad , \quad |v| = 10$

3) Find angle between u & v.

$\cos\theta = \dfrac{u\cdot v}{|u||v|} \Rightarrow \cos\theta = \dfrac{0}{50} \qquad \cos\theta = 0$

$\theta = 90°$

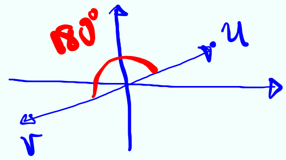
If $u\cdot v = 0 \Rightarrow u \perp v$

$\theta = 90°$

Nov 19-11:04 AM

$u = \langle 4, 3 \rangle \quad v = \langle -4, -3 \rangle$

1) Draw u & v in standard position.



2) Find $|u|$, $|v|$, and $u \cdot v$.

$|u| = 5 \quad |v| = 5$

$= \langle 4,3 \rangle \cdot \langle -4,-3 \rangle$

$= -25$

3) Find angle between them

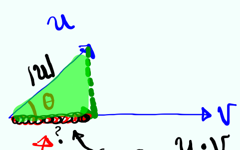
$\cos\theta = \dfrac{u \cdot v}{|u|\,|v|} = \dfrac{-25}{5 \cdot 5}$

$\cos\theta = -1$

$\theta = \cos^{-1}(-1)$

$\theta = 180°$

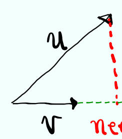
Nov 19-11:11 AM



$\cos\theta = \dfrac{u \cdot v}{|u|\,|v|}$

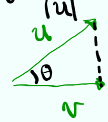
Scalar Proj of u onto v

$\text{Proj}_v u = (\ ?\ )\vec{v}$



need to extend V

$\cos\theta = \dfrac{?}{|u|} \qquad ? = |u|\cos\theta$



$\text{Proj}_v u = |u|\cos\theta \dfrac{1}{|v|}\vec{v}$

$= \cancel{|u|}\dfrac{u \cdot v}{\cancel{|u|}|v|}\dfrac{1}{|v|}\vec{v}$

$= \dfrac{u \cdot v}{|v|^2}\vec{v}$

$\text{Proj}_v u = \dfrac{u \cdot v}{v \cdot v}\vec{v}$

Scalar Proj of u onto v.

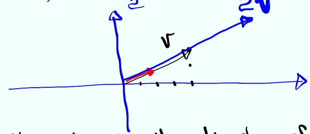
Nov 19-11:17 AM

$v = \langle 4, 2 \rangle$

Graph $2\vec{v}$, and $\frac{1}{2}v$

$2\vec{v} = \langle 8, 4 \rangle$

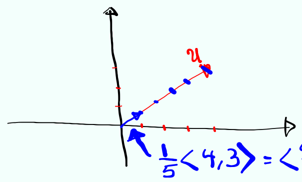
$\frac{1}{2}v = \langle 2, 1 \rangle$



Find a unit Vector in the direction of $\langle 4, 3 \rangle$

u

$|u| = 5$



$\frac{1}{5}\langle 4, 3 \rangle = \langle \frac{4}{5}, \frac{3}{5} \rangle$

To find a unit vector in the direction of $u = \langle a, b \rangle$, multiply by $\frac{1}{|u|}$

$u = \langle -2, 4 \rangle$

$|u| = \sqrt{4 + 16}$

$= \sqrt{20} = 2\sqrt{5}$

$\boxed{\frac{1}{2\sqrt{5}}\langle -2, 4 \rangle}$

unit Vector in same direction of $\langle -2, 4 \rangle$

$\frac{\sqrt{5}}{5}\langle -1, 2 \rangle$

Nov 19-11:24 AM